Journal of Economics and Behavioral Studies
Vol. 6, No. 11, pp. 840-847, November 2014 (ISSN: 2220-6140)

A Simulation Approach to Determine the Probability of Demand during Lead-Time When Demand Distributed Normal and Lead-Time Distributed Gamma

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Abstract: Globalization and advances in information and production technologies make inventory management can be very difficult even for organizations with simple structures. The complexities of inventory management increase in multi-stage networks, where inventory appears in multiple tiers of locations. Due to massive practical applications in the reality of the world, an efficient inventory system policy whether single location or multi-stage location will avoid falling into overstock inventory or under stock inventory. However, the optimality of inventory and allocation policies in a supply chain is still unknown for most types of multi-stage systems. Hence, this paper aims to determine the probability distribution function of demand during lead-time by using a simulation model when the demand distributed normal and the lead-time distributed gamma. The simulation model showed a new probability distribution function of demand during lead-time in the considered inventory system, which is, Generalized Gamma distribution with 4 parameters. This probability distribution function makes the mathematical expression more difficult to build the inventory model especially in multistage or multi-echelon inventory model.

Keywords: *Normal demand, Gamma lead-time, forecasting, simulation*

1. Introduction

The main problem in inventory models is the determination of the probability distribution function of demand during lead-time when demand and lead-time are subject to a probabilistic distribution function. When demand is uncertain and probabilistic it's difficult to predictor the demand quantities especially in fast moving items. However, the treatment in inventory models is different between fast moving items (high demand) and slow moving items like spear parts or stock out. This paper focused on fast moving items and has been applied in cement industry in Iraq/ Kurdistan region. The data is for three years period from 2011 till 2013, which face a very high and probabilistic demand and lead-time. The previous studies that considered probability of demand and lead-time, normal distribution often has been used as a demand model in fast moving items (Axsater, 2006; Bagchi, Hayya, 1984; Dekker, Kleijn& De Kok, 1998; Lee, 2005; Wu, Lee, & Tsai, 2007). And others adopt Poisson or compound Poisson as a demand distribution and mostly constant, fixed or neglected lead-time (Axsäter & Marklund, 2008; Axsäter, 1984; Clark & Scarf, 1960; Graves, 1986; Hausman & Erkip, 1994; Hosoda & Disney, 2006; Muckstadt, 1986; Ravichandran, 1995; Saffari & Haji, 2009; Sherbrooke, 1968; Zhao, Zhan, Huo, & Wu, 2006).Generally, these assumptions are valid in inventory models that carry expensive items and face low demand but highly uncertain demand (Caglar, Li & Simchi-Levi, 2004; Graves, 1985; Muckstadt, 1973; Sherbrooke, 1968) that's because of the stationary and stability of the market demand and it's not subject to sudden changes like politic factors and Security Circumstances as in our case. Bagchi and Hayya(1984) modify mathematical expressions for potential seals depend on demand distributed normal and lead-time distributed Erlang. Baten & Kamil (2009) used 2-weibull distribution parameters when demand distributed Weibull and deterministic lead-time. Our purpose in this paper is to determine the probability distribution function of demand during lead-time depend on a simulation approach by using forecasting method (exponential smoothing method) for the demand and the probability distribution function of the lead-time.

2. Forecasting Method

Forecasting is the estimate values of the variables for cases do not fall within the available observation units. Forecasting is not intuitively or conjectures, but it is the statistical treatment of the past data to give any estimate of the variables state in the future. The predictive study may indicate for example, a large unexpected rise in consumption, which means the possibility of increasing the size of the stock and then lead to increase items or products inventory to face the increase in consumption, or the predictive study may indicate about the possibility of the economic crisis occurrence, therefore it is necessary take procedures and follow the needed policies to avert a crisis and ward off risks. However, the interest of forecasting in inventory policies is due to:

- The change requirements which the variable rate is not revolve around constant average, but if the requirements fluctuated around a constant average can calculate the reserve to face this fluctuation.
- The length of lead-time, i.e. the required period to access the orders and then the possibility of achieving a balance between compensation and consumption, unless their predictions of the expected consumption during upcoming period. During the long period of the orders arrive, that may arise the changes in consumption case, this means if there were not forecasts about consumption during this period the balance will not be achieved between compensation and consumption.
- the sudden changes in the volume of consumption, which are mostly due to the unusual circumstances which naturally don't allow to estimated exactly, like the sudden increase in the income or contribute as much as more in the provision of services.

In most of the literatures, forecasting addressed to estimate the model parameters or knowing the demand distribution. Most of the time, forecasting is the related tool with simulation (Baykal-Gurosy & Erkip, 2010; Choi, Chiu, & Fu, 2011; Wang, 2009; Wang & Lin, 2010). And the most often method (means and standard deviation) of demand formula are provided in a wide variety of exponential smoothing methods (Snyder, Koehler, Hyndman, & Ord, 2004). These two factors (means and standard deviation) depend on the effects of the data trends and seasonal, i.e. the fluctuations in the behaviors of demand data can be stationary over the time, increase trends over the time, decrease trends over the time, or seasonal. Exponential smoothing methods was introduced firstly by (Brown, 1959). A new technique for forecasting inventory policy presented by using multi regression based forecasting models for predicting the supplier the total profit in two-echelon SC. The assumed model are build according to weighting elements method and transformation of data confer higher predictive precision than traditional regression models

Exponential smoothing method: The Exponential smoothing method is the most statistical methods used in the field of inventor control. The reason for using this method is due to

- Easy method for calculation.
- It is sensitive with the variables in any time.
- No need to store large amounts of information.

The statistical principle of forecasting for a variable based on the variable patterns composed of two parts. First, inevitable regular free from vibrations and can be expressed in equation. Second, probabilistic random variable has a certain distribution, with zero average and a certain standard deviation ($\sigma\epsilon 2$). If we denote the first part as U_t, and second part ϵt then the expression become

$$X_t = U_t + \varepsilon_t \,, t = 1,2,3, \ldots.. \qquad (1)$$

If U_t reflect the fixed amount ($Ut = a$), this means that the value of the variable consists of a constant plus random variable.

$$X_t = a + \varepsilon_t \qquad (2)$$

In the case of availability of a series values of the variables, this series has fixed average fluctuating around up and down, and for the purpose of forecasting the value of exponential smoothing the original equation of exponential smoothing is from (Brown, 1959)

$$F_{t+1} = aX_t + (1 - \alpha)F_t \qquad (3)$$

Where

F_{t+1}: the forecasted observation value for next period.

X_t: current period observation value.

F_t: the forecasted observation value for current period.

α: constant smoothing, ranging between zero and 1, $0 \leq \alpha \leq 1$

This method is used when the data is stationary, where there is no seasonal pattern or periodic. From equation (3), we can note that, the new forecasted value of F_{t+1} subject to

- The current observation with weighed α
- The current period predicted with weighed (1- α).

This method is named exponential smoothing (ES) because the meaning of F_t is clearer after degenerate to its compounds as in equation (3).

$$F_t = \alpha X_{t-1} + (1-\alpha)F_{t-1} \quad (4)$$

$$F_{t+1} = \alpha X_t + (1-\alpha)\{\alpha X_{t-1} + (1-\alpha)F_{t-1}\} \quad (5)$$

$$F_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + (1-\alpha)^2 F_{t-1} (6)$$

Similar, we degenerate F_{t-1} to its compounds

$$F_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + (1-\alpha)^2 X_{t-2} + (1-\alpha)^3 X_{t-3+} \dots\dots\dots\dots. + (1-\alpha)^{N-1} X_{t-(N-1)} \quad (7)$$

We note from equation (7) the effect of the previous observation less exponentially with the time. i.e. the given weighting for each observation from the last observation will be less exponentially.

The problem now is how to calculate the first forecasted value F_t? This is because in equation (3) the value of F_{t+1} depend on X_t and F_t. For example, $F_2 = \alpha X_1 + (-\alpha) F_1$. To calculate the value of (F_1) there are more than a direction or idea:

- The initial value of (F_1) is the average of real observation or historical data of (X_t)

$$F_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (8)$$

  This technique used in situations when exponential smoothing need quickly and not use for the future researches.

- The initial value of (F_1) is the same value for the real observation (X_1). This technique used when the data is not adequate and convergent.

$$F_1 = X_1 \quad (9)$$

- The initial value of (F_1) is the average of the first quarter of the real observation (X_t). This technique used for the future studies and researches (long-term) as we adopted in this paper.

$$F_1 = \frac{1}{(\frac{n}{4})}\sum_{i=1}^{\mathrm{Int}\,(\frac{n}{4})} X_i \quad (10)$$

Constant smoothing: The value of smoothing constant has an effective impact to success the forecasting model. The value of smoothing constant (α) ranging between zero and 1, $0 \le \alpha \le 1$, this value depends on the amount of weight that will give to the new experience, the highest value is maximize the weight, reference to the equation (3) if (α=1) this means we gave all weight to the last value and ignore the old average. If (α=0.5) gives an indication of attaches great importance to last real consumption. If (α=0) means ignore the last value. As conclusion, the high smoothing constant leads to the forecasting model is fast response but non-stationary, otherwise, the low smoothing constant leads to slow response but stationary forecasting model. To determine the smoothing constant value, we can use forecast error as a measurement to choose the smoothing constant. The forecast error has zero average and standard deviation (σ_e).

$$e_t = X_t - F_t \quad (11)$$

Since the forecast error can be negative or positive, to solve this problem take the value square. And for the purpose of obtaining a uniform scale with other data, this require to calculate "mean sum Square Error" and since its unit measurement is the square units of original values, it is possible to calculate a new measure by finding the square root of Mean Square Error (MSE).

$$MSR = \sqrt{\frac{1}{n}\sum e_i^2} \quad (12)$$

Based on this, the smoothing constant altered within the range of reasonable, then chooses the constant that lead to less (MSR).

Forecasting Error: Irrespective of which method used to get the forecasting value of (X) for the period or next periods, the real value will remain different with predicted values somewhat either plus or minus. In order to reach out to estimate this error we use the so-called forecasting error, where, forecasting error is the different between the real value (X_t) and the forecast value (F_t) as in equation (11). In order to reach a predictive values equal to the real values, the differences between these two values must equal to zero. But the reality is not so, because the value of (e_t) cannot be known previously unless knowing the value of (F_t). It has been found that the forecasting error distributed Normally (Brown, 1959). To determine the parameters of this distribution must know the (mean and variance (therefore, the standard deviation)). The variance error for the forecasting model exponential smoothing (ES) is: $\sigma_e^2 = \frac{2}{2-\sigma}\sigma_\varepsilon^2$ (13)

Where, σ_ε^2 is the variance of random variable, and to calculate the variance error (σ_e^2) from equation (13) must specify (σ_ε^2), the latter also is unknown. So, we will use the numerical relationship between (mean absolute deviation (MAD)) and standard deviation to estimate the value of σ_ε^2.

$$\sigma_t = 1.25\text{MAD}_t \qquad (14)$$

Where, MAD= sum of absolute error / number of error (15)

Most facilities, manufactories, and stakeholders prefer to use mean absolute deviation to obtain the standard deviation for the following reasons:

- Easy method to calculate.
- The mean absolute deviation can fit the exponential smoothing method, as it can get the estimate of mean absolute deviation by using exponential smoothing method.

$$MAD_{t+1} = \alpha|e_t| + (1-\alpha)MAD_t \qquad (16)$$

- Mean absolute deviation is the estimation of the forecasting error for next period.
- Mean absolute deviation is the average amount of stock out, if doesn't use the safety stock.
- Mean absolute deviation is the median value of convergence or divergence, when using suitable forecasting model.

From equation (16), the problem is how to find the initial value of (MAD)? To find the initial value of (MAD_1), we follow the same steps to find (F_1) in section (2.1), but here we take the absolute error.

Probability of demand during lead-time: Demand during lead-time is the joint distribution of demand distribution and lead-time distribution depending on the parameters (mean and standard deviation) for each of them. The question that arises is why demand during lead-time is very important in inventory control? The answer of this question is, when the items, goods, or products nearing completion the decision maker start to make a request an order quantity to meet the needs of consumers and not to fall in the shortage. Certainly during this period and until arrive the required quantity to the depot, customer demand is continuous, and since the processes very nested, it is difficult to record these data of demand through the items or products arrival. Where, if the items arrived late to the warehouse, the warehouse covers or satisfies the customer demands or the markets demands from the safety stock (inventory on hand). Here the aim is not how much or when we order, the aim is the demand is probabilistic and the arrival time of the items until placed in the warehouse is also probabilistic (lead-time), therefore, we need to know the probability distribution function of demand during lead-time, to deal with it according to the requirements the concerned party. The mathematical approach to find (mean and standard deviation) of demand during lead-time can be calculated from the following two equations (Fishman, 1973)

$$\mu_L = E(X)E(L) \qquad (17)$$

$$\sigma_L = \sqrt{E(X)Var(X) + [E(X)]^2 Var(L)} \qquad (18)$$

Where, E(x) is the expected demand, var(x) is the variance of demand, E(L) the expected lead-time, and var(L) the variance of lead-time, μ_L, σ_L is the mean and variance of demand during lead-time respectively. But these two equations give only the value of (mean and variance) without knowing the probability distribution function (pdf) the importance.

3. Simulation approach

Most of the time simulation is only the way to know the probability distribution function of demand during lead-time. The aim of using simulation is to generate data depend on demand and lead-time distribution. But before starting we should know the probability distribution function of the lead-time. After using "Easy Fit" software to test the lead-time data, the results shows the lead-time distributed Gamma with shape parameter (α =1.3484134) and scale parameter (β =4.4800435) as Kolmogorov-Smirnov test with P-value=0.985 see figure 1 and 2.

Next step is finding the mean and the standard deviation of the demand data. The suitable method to find the mean and the standard deviation when the demand data fluctuations are normal and stationary around a certain mean is exponential smoothing method. See figure 3.The mean and the standard deviation can be calculated for one period (F_{t+1} and MAD_{t+1}) as in equation (3) and (16) respectively with smoothing constant (α=0.1), the best value of (α) which gave the less value of mean square error MSR. The results shows the mean (μ=12037.73) and the standard deviation (σ =3236.093).

Figure 1: Screen of the data analysis

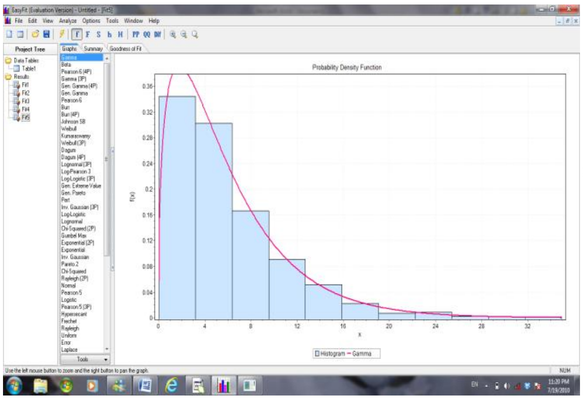


Figure 2: the Probability density function of the lead-time

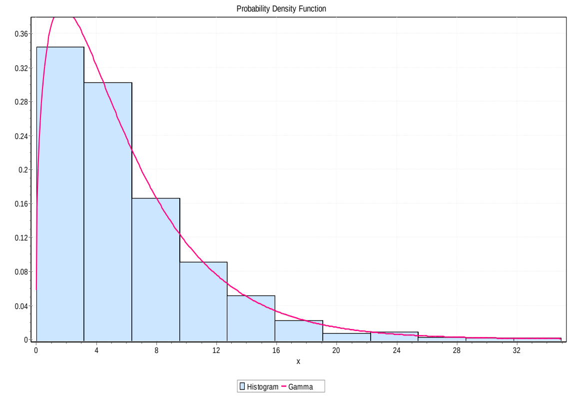
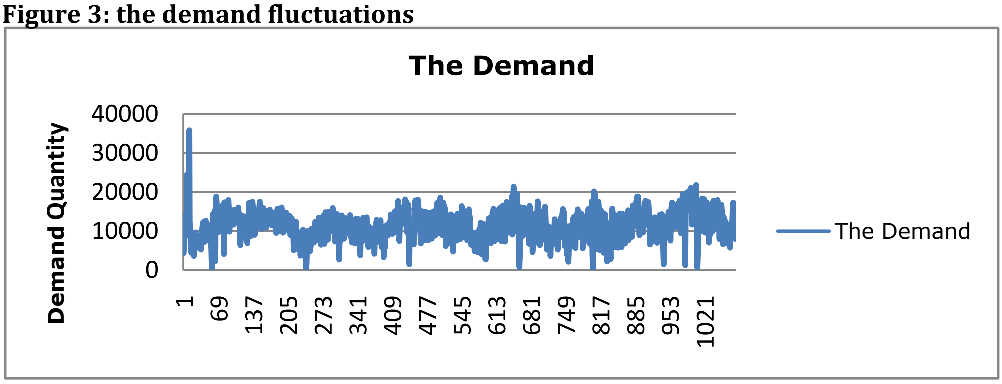


Figure 3: the demand fluctuations



After the completion of knowing the statistical distribution of the demand and the lead-time, we start with generating data process of demand during lead-time. To generate these data we must use a simulation approach. Where, prepare a program method for this purpose as follow: Simulation program generates random number not less than 1500 observations depending on the lead-time distribution parameters as we calculated. Then generating 1500 random number for the demand distribution depending on (mean and standard deviation) extracted from the exponential smoothing method. To shows the demand during lead-time, we merging the two distributions of demand and lead-time. In other words, when generating lead-time data for 1500 random number we will generate inside each value of these number data distributed as demand distribution within (mean and standard deviation). The following example illustrate the idea better, let suppose the initial generated value of lead-times is (6.57), then generating values distributed according to the demand distribution as much as this value and collected. The result of this process represent the demand during lead-time, and repetitive this procedures as much as the lead-time data generated (1500) times, Consequently get 1500 observation of demand during lead-time. Figure (4) shows the flow chart of generating data of demand during lead-time.

4. The simulation results

After we generated the data by the simulation program, know we test these data by using one of the ready statistical software (SPSS, Statistica, Easy Fit, statgraph, etc.) to know the probability distribution function of demand during lead-time. To ensure the validity and the credibility of the generated data of demand during lead-time, some tests should be done as follows:

- Extracting the mean and the standard deviation of the generated demand during lead-time. Where, the mean (μ=5.953) and the standard deviation (σ=25.916)

- Calculate the mean the standard deviation for the demand during lead-time regardless of the statistical distribution from equation (17) and (18). Where, the calculated mean from equation (17) is (μ=6.040) and the standard deviation from equation (18) is (σ=27.063).
- Comparing the results of point 1 and 2, if the results are close this mean the statistical distribution of the generated data is correct, otherwise, there is an error in the simulation procedures.

After testing the generated data by one of the ready software as we mentioned above, for example “Easy Fit” software, the analysis shows that the demand during lead-time probability distribution is Generalized Gamma distribution with 4 parameters (K-continuous shape parameter (K>0), α -continuous shape parameter (α>0), β -continuous scale parameter (β >0) and γ-continuous location parameter as Kolmogorov-Smirnov test with P-value = 0.93584. See figure5 and 6.

Figure 4: the demand during lead-time generating framework

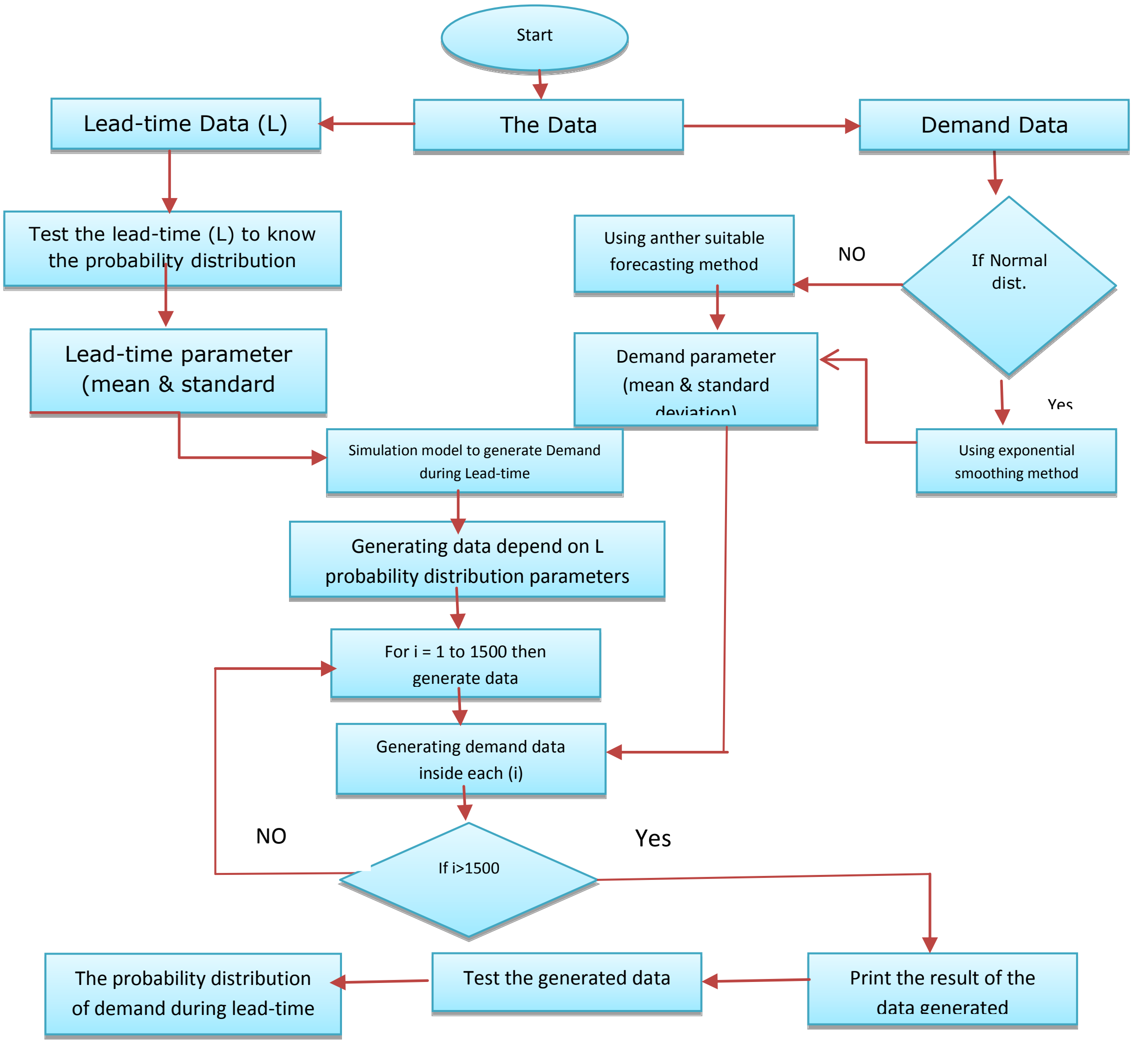


Figure 5: Screen of the data analysis during lead-time

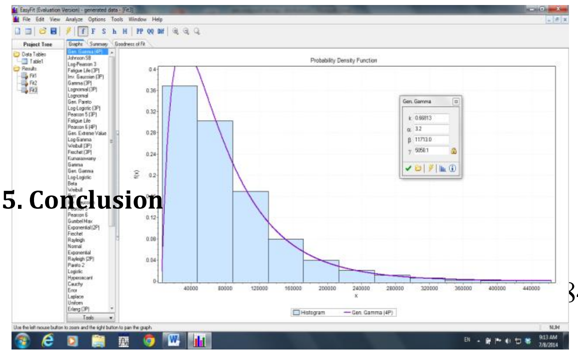
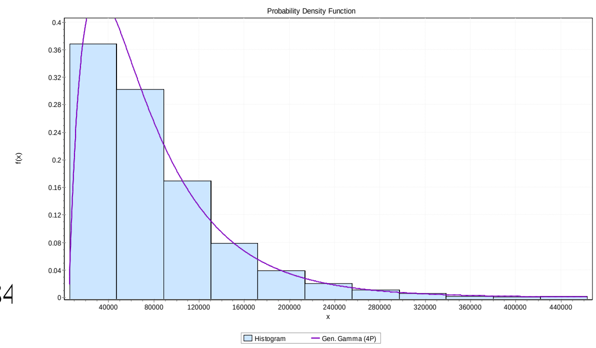


Figure 6 the Probability density function of demand



5. Conclusion

This paper focused on how to find or determine the probability distribution function of demand during lead-time by a simulation model when the demand distributed normal and the lead-time distributed Gamma. In the most of the literature, we found the demand is constant or stochastic (normal, Poisson or compound Poisson) and the lead-time is constant, fixed, zero or neglected. The inventory models treatment is not complex as to be both of them probabilistic. Also, the literature that supposed the lead-time is probabilistic often it was normal distribution and sometimes Poisson, Weibull or Erlang distribution. These assumptions are valid in inventory system when the market is stabile and it is not subject to any sudden changes likes the politic factors and Security Circumstances as in this paper. The data analysis in this paper shows a new probability distribution function in inventory models, which is Generalized Gamma distribution with 4 parameters which make the mathematical expression of the inventory models more complex to find the inventory model parameters or variables. This leads us to solve or derivation a new mathematical model in inventory system which is very complex.

References

Axsater, S. (2006). A simple procedure for determining order quantities under a fill rate constraint and normally distributed lead-time demand. *European Journal of Operational Research*, 174, 480–491. doi:10.1016/j.ejor.2005.01.037.

Axsäter, S. (1984). In-Process safety stock. In 23rd conference on Decision and Control (pp. 839–842).

Axsäter, S. & Marklund, J. (2008). Optimal Position-Based Warehouse Ordering in Divergent Two-Echelon Inventory Systems. *Operations Research*, *56*(4), 976–991. doi:10.1287/opre.1080.0560.

Bagchi, U. & Hayya, J. (1984). Demand during Lead Time for Normal Unit Demand and Erlang Lead Time. *Operations Research Society1*, 35(2), 131–135.

Baten, M. A. & Kamil, A. (2009). An Optimal Control Approach to Inventory-Production Systems with Weibull Distributed Deterioration. *Mathematics and Statisitcs*, 5(3), 206–214.

Baykal-Gurosy, M. & Erkip, N. (2010). FORECASTING FOR INVENTORY PLANNING UNDER CORRELATED. Wiley Encyclopedia of Operations Research and Management Science, 1–11.

Brown, R. G. (1959). Statistical forecasting for inventory control. McGraw-Hill, New York.

Caglar, D., Li, C. L. & Simchi-Levi, D. (2004). Two-echelon spare parts inventory system subject to a service constraint. *C IIE Transactions*, 36(7), 655–666.

Choi, T. M., Chiu, C. H. & Fu, P. L. (2011). Periodic Review Multiperiod Inventory Control Under a Mean–Variance Optimization Objective. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *41*(4), 678–682. doi:10.1109/TSMCA.2010.2089515

Clark, A. & Scarf, H. (1960). Optimal policies for a Multi-echelon inventory problem. *Management Science*, 6(4), 475–490.

Dekker, R., Kleijn, M. J. & De Kok, A. G. (1998). Dekker, R., Kleijn, M. J., & De Kok, A. G. (1998). The break quantity rule's effect on inventory costs in a 1-warehouse,< i> N</i>-retailers distribution system. *International Journal of Production Economics*, 56, 61–68.

Fishman, G. S. (1973). Concepts and methods in discrete event digital simulation (pp. l60–168). New York: Wiley.

Graves, S. (1985). A Multi-Echelon Inventory Model For A Repairable Item With One-For-One Replenishment. *Management Science*, 31(10), 1247–1256.

Graves, S. (1986). A Multi-Echelon Inventory Model For A Repairable Item With One-For-One R. *Managenetn Science*, 31(10), 1247–1256.

Hausman, W. & Erkip, N. (1994). Multi-echelon vs . single-echelon inventory control policies. *Management Science*, 40(5), 597–602.

Hosoda, T. & Disney, S. M. (2006). On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. *The International Journal of Management Science*, 34, 344–358. doi:10.1016/j.omega.2004.11.005

Lee, W. (2005). Inventory model involving controllable backorder rate and variable lead time demand with the mixtures of distribution. *Applied Mathematics and Computation*, 160, 701–717. doi:10.1016/j.amc.2003.11.039.

Muckstadt, J. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science*, 20(4 part1), 472–81.

Muckstadt, J. (1986). A Model For A Multi-Item , Multi-Echelon , Multi-Indenture Inventory System. *Management Science*, 20(4), 472–781.

Ravichandran, N. (1995). Theory and Methodology Stochastic analysis of a continuous review perishable inventory system with positive lead time and Poisson demand. *European Journal of Operational Research*, 84(93), 444–457.

Saffari, M. & Haji, R. (2009). Queueing system with inventory for two-echelon supply chain. In *2009 International Conference on Computers & Industrial Engineering*, 2, 835–838). Ieee. doi:10.1109/ICCIE.2009.5223801.

Sherbrooke, C. (1968). METRIC: a mulri-Echelon technique for recoverable item control. *Operations Research*, 16(1), 122–141.

Snyder, R., Koehler, A., Hyndman, R. & Ord, J. K. (2004). Exponential smoothing models: Means and variances for lead-time demand. *European Journal of Operational Research*, 158(2), 444–455. doi:10.1016/S0377-2217(03)00360-6.

Wang, C. (2009). Using the third order exponential smoothing forecasts on reducing the bullwhip effect and inventory costs in supply chain. In 2009 IEEE International Conference on Grey Systems and Intelligent Services (GSIS 2009) (pp. 1612–1617). Ieee. doi:10.1109/GSIS.2009.5408172.

Wang, K. J., Bunjira, M. & Lin, Y. S. (2010). Forecasting based inventory management for supply chain. In 2010 Seventh International Conference on Fuzzy Systems and Knowledge Discovery (pp. 2964–2967). Ieee. doi:10.1109/FSKD.2010.5569080.

Wu, J., Lee, W. & Tsai, H. Y. (2007). Computational algorithmic procedure of optimal inventory policy involving a negative exponential crashing cost and variable lead time demand. *Applied Mathematics and Computation*, 184, 798–808. doi:10.1016/j.amc.2006.05.202

Zhao, D., Zhan, Y. A. N., Huo, Y. & Wu, X. (2006). Lot-Sizing Disposal Model In Reverse Logistics. In the Fifith International Conference on Machine Learning and cybernetics (13–16).